

Scalar Field Dark Energy Parametrization

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We propose a new Dark Energy parametrization based on the dynamics of a scalar field. We use an equation of state $w = (x - 1)/(x + 1)$, with $x = E_k/V$, the ratio of kinetic energy $E_k = \dot{\phi}^2/2$ and potential V . The eq. of motion gives $x = (L/6)(V/3H^2)$ and with a solution $x = ([1 + 2L/3(1 + y)]^{1/2} - 1)(1 + y)/2$ where $y \equiv \rho_m/V$ and $L \equiv (V'/V)^2(1 + q)^2$, $q \equiv \ddot{\phi}/V'$. Since the universe is accelerating at present time we use the slow roll approximation in which case we have $|q| \ll 1$ and $L \simeq (V'/V)^2$. However, the derivation of L is exact and has no approximation. By choosing an appropriate ansatz for L we obtain a wide class of behavior for the evolution of Dark Energy without the need to specify the potential V . In fact w can either grow and later decrease, or other way around, as a function of redshift and it is constraint between $-1 \leq w \leq 1$ as for any canonical scalar field with only gravitational interaction. Furthermore, we also calculate the perturbations of DE and since the evolution of DE is motivated by the dynamics of a scalar field the homogenous and its perturbations can be used to determine the form of the potential and the nature of Dark Energy. Since our parametrization is on L we can easily connect it with the scalar potential $V(\phi)$.

I. INTRODUCTION

In the last years the study of our universe has received a great deal of attention since on the one hand fundamental theoretical questions remain unanswered and on the other hand we have now the opportunity to measure the cosmological parameters with an extraordinary precision. Existing observational experiments involve measurement on CMB [1] or large scale structure LSS [2] or supernovae SN1a [3], and new proposals are carried out [4]. It has been established that our universe is flat and dominated at present time by Dark Energy "DE" and Dark Matter "DM" with $\Omega_{DE} \simeq 0.73$, $\Omega_{DM} \simeq 0.27$ and curvature $\Omega_k \simeq -0.012$ [1]. However, the nature and dynamics of Dark Energy is a topic of mayor interest in the field [5]. The equation of state "EOS" of DE is at present time $w_o \simeq -0.93$ but we do not have a precise measurements of $w(z)$ as a function of redshift z [1]. Since the properties of Dark Energy are still under investigation, different DE parametrization have been proposed to help discern on the dynamics of DE [6]-[12]. Some of these DE parametrization have the advantage of having a reduced number of parameters, but they may lack of a physical motivation and may also be too restrictive. Furthermore, the evolution of DE may not be enough to distinguish between different models and the perturbations of DE may be fundamental to differentiate them.

Perhaps the best physically motivated candidates for Dark Energy are scalar fields which can be minimally coupled, only via gravity, to other fluids [11, 12] or can interact weakly in interacting Dark Energy "IDE" [15, 16]. Scalar fields have been widely studied in the literature [11, 12] and special interest was devoted to tracker fields [11] since in this case the behavior of the scalar field ϕ is very weakly dependent on the initial conditions at a very early epoch and well before matter-radiation equality. In this class of models the fundamental question of why DE is relevant now, also called the coincidence problem, can be ameliorated by the insensitivity of the late time dy-

namics on the initial conditions of ϕ . However, at present time we are not concern at this stage with the initial conditions but we work from present redshift $z = 0$ to larger values of z in the region where DE and its perturbations are relevant. In this case the conditions for tracker fields do not necessarily apply. Interesting models for DE and DM have been proposed using gauge groups, similar to QCD in particle physics, and have been studied to understand the nature of Dark Energy [13] and also Dark Matter [14].

Here we propose a new DE parametrization based on scalar fields dynamics, but the parametrization of w can be used without the connection to scalar fields. This parametrization has a reach structure that allows w to have different evolutions and it may grow and later decrease or other way around. We also determine the perturbations of DE which together with the evolution of the homogenous part can single out the nature of DE. With the underlying connection between the evolution of w and the dynamics of scalar field we could determine the potential $V(\phi)$. The same motivation of parametrizing the evolution of scalar field was presented in an interesting paper [10]. We share the same motivation but we follow a different path which has the same number of parameters but it has a richer structure and it is easier to extract information on the scalar potential $V(\phi)$.

We organized the work as follows: in Sec.IA we give a brief overview of our DE parametrization. In Sec.II we present the dynamics of a scalar field and the set up for our DE parametrization presented in Sec.III. We calculate the DE perturbations in Sec.IV and finally we conclude in Sec.V.

A. Overview

We present here an overview of our w parametrization. The EOS is

$$w = \frac{p}{\rho} = \frac{x-1}{x+1}. \quad (1)$$

with $x \equiv E_k/V$ the ratio of kinetic energy and potential. The equation of motion of the scalar field gives (c.f. eq.(17)),

$$x = \frac{\left(\sqrt{1 + \frac{2L}{3(1+y)}} - 1\right)(1+y)}{2}. \quad (2)$$

where $L = (V'/V)^2 A$, $y = \rho_m/V$ the ratio of matter and V and $A \equiv (1+q)^2$, $q \equiv \ddot{\phi}/V'$. Eq.(2) is an exact equation and is valid for any fluid evolution and/or for arbitrary potentials $V(\phi)$.

The aim of our proposed parametrization for L, y is to cover a wide range of DE behavior. Of course other interesting parameterizations are possible. From the dynamics of scalar fields we know that the evolution of w close to present time is very model dependent. For example, in the case of $V = V_o \phi^{-2/3}$, used as a model of DE derived from gauge theory [13], the shape of $w(z)$ close to present time depends on the initial conditions and it may grow or decrease as a function of redshift z . Of course if we change the potential the sensitivity on the choice of V and initial conditions will vary a lot. We also know that tracker fields are attractor solutions but in most cases they do not give a negative enough w_o [11]. The dynamics of scalar fields with a single potential term, for a wide class of models, gives an accelerating universe only if $\lambda = V'/V \rightarrow 0$ or to a constant $|\lambda|$ with $w = -1 + \lambda/3$ [12]. In this class of models the EOS, regardless of its initial value, goes to a period of kinetic domination where $w \simeq 1$ and later has a steep transition to $w \simeq -1$, which may be close to present time, and finally it grows to w_o in a very model and initial condition dependent. Furthermore, if instead of having a single potential term we have two competing terms close to present time, the evolution of $w(z)$ would even be more complicated. Therefore, instead of deriving the potential V from theoretical models as in [13] we propose to use an ansatz for the functions L, y which on the one hand should cover as wide as possible the different classes of DE behavior with the least number of parameters, without sacrificing generality, and on the other hand we like to have the ansatz as close as possible to the known scalar field dynamics. We believe that using our model will greatly simplify the extraction of DE from the future observational data. We propose therefore the ansatz (c.f. eq.(41))

$$L = L_o + L_1 y^\xi f(a) = L_o + L_1 y_o^\xi \left(\frac{a^{3\xi w_o}}{1 + (a/a_t)^k} \right) \quad (3)$$

$$f(a) = \frac{1}{1 + (a/a_t)^k} = \frac{1}{1 + [(1+z_t)/(1+z)]^k} \quad (4)$$

where L_o, L_1 are free parameters giving w_o and $w_1 = w(z \gg z_t)$ at early times, $f(z)$ is a function that goes from $f(z=0) = 1/(1+(1+z_t)^k)$ at $z=0$ to $f(z \gg 1) = 1$ and z_t sets the transition redshift between w_o and w_1 (a subscript o represents present time quantities) while k the steepness of the transition and ξ takes only two values $\xi = 1$ or $\xi = 0$. We show that a steep transition of w has a bump in the adiabatic sound speed c_a^2 which could be detected in large scale structure. Since the universe is accelerating at present time we may take the slow roll approximation where $|q| \ll 1$, $A \simeq 1$ and $L \simeq (V'/V)^2$. However, the derivation of L in eq.(2) is exact and has no approximation. We will show in section III that w can have a wide range of behavior and in particular it can decrease and later increasing as a function of redshift and viceversa, i.e. the shape and steepness are not predetermined by the choice of parametrization. Of course we could use other parametrization since the evolution of x and w in eqs.(1) and (2) are fully valid. There is also no need to have any reference to the underlying scalar field dynamics, i.e. it is not constraint to scalar field dynamics. However, it is when we interpret $x \equiv \dot{\phi}^2/2V$ with $L = (V'/V)^2 A$ and the ratio $y = \rho_m/V$ that we connect the evolution of w to the scalar potential $V(\phi)$.

II. SCALAR FIELD DYNAMICS

We are interested in obtaining a new DE parametrization inferred from scalar fields. Since it is derived from the dynamics of a scalar field ϕ we can also determine its perturbations which are relevant in large scale structure formation. We start from the equation of motion for a canonical scalar field $\phi(t, x)$ with a potential $V(\phi)$ in a FRW metric. The homogenous part of ϕ has an equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad (5)$$

where $V' \equiv dV/d\phi$, $H = \dot{a}/a$ is the Hubble constant, a is the scale factor with $a_o/a = 1+z$ and a dot represents derivative with respect to time t . Since we are interested in the epoch for small z we only need to consider matter and DE and we have

$$3H^2 = \rho_m + \rho_\phi \quad (6)$$

in natural units $8\pi G = 1$. The energy density ρ and pressure p for the scalar field are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V, \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V \quad (7)$$

and the equation of state parameter "EOS" is

$$w \equiv \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} = \frac{x-1}{x+1} \quad (8)$$

where we have defined the ratio of kinetic energy and potential energy as

$$x \equiv \frac{\dot{\phi}^2}{2V} \quad (9)$$

The value of x gives w or inverting eq.(8) we have $x = (w+1)/(w-1)$. The evolution of $x(z) \geq 0$ determines the Dark Energy w in the range $-1 \leq w \leq 1$. For growing x the EOS w becomes larger and at $x \gg 1$ one has $w \simeq 1$ while a decreasing x has w approaching -1 for $x = 0$.

In terms of x and $y \equiv \rho_m/V$ we have

$$\rho_\phi = V(x+1), \quad p_\phi = V(x-1), \quad \rho_m = Vy \quad (10)$$

and

$$\Omega_\phi = \frac{1+x}{1+x+y}, \quad \Omega_m = \frac{y}{1+x+y}, \quad (11)$$

$$3H^2 = \rho_m + \rho_\phi = V(1+x+y) \quad (12)$$

We can write

$$x \equiv \frac{\dot{\phi}^2}{2V} = \frac{V}{3H^2} \frac{V'^2}{6V^2} (1+q)^2 = \frac{V}{3H^2} \frac{L}{6} \quad (13)$$

using

$$\dot{\phi} = -\frac{V' + \ddot{\phi}}{3H} = -\frac{V'(1+q)}{3H} \quad (14)$$

and we defined

$$L \equiv \left(\frac{V'}{V}\right)^2 (1+q)^2 = \lambda^2 A, \quad (15)$$

$$\lambda \equiv \frac{V'}{V}, \quad q \equiv \frac{\ddot{\phi}}{V'}, \quad A \equiv (1+q)^2$$

with $q > -1$. Since the r.h.s. of eq.(13) still depends on x through H we use eq.(12) and eq.(13) becomes then

$$x = \frac{L}{6(1+x+y)} \quad (16)$$

which has a simple solution

$$x = \frac{\left(\sqrt{1 + \frac{2L}{3(1+y)^2}} - 1\right)(1+y)}{2}. \quad (17)$$

Eq.(17) sets our DE parametrization as a function of L and y . For small q and A is close to one and the quantity L gives direct information on the potential and its derivative.

1. Dynamical evolution of x and y

Differentiating x and $y \equiv \rho_m/V$ w.r.t. time we get the evolution

$$\dot{x} = \frac{\dot{\phi}}{V} (\ddot{\phi} - xV') = \frac{\dot{\phi}V'}{V} (q - x) \quad (18)$$

$$= 6Hx \left(\frac{x-q}{1+q}\right) \quad (19)$$

and

$$\dot{y} = \frac{\dot{\rho}_m}{V} - \frac{yV'\dot{\phi}}{V^2} = 3Hy \left(\frac{2x-q-1}{1+q}\right) \quad (20)$$

were we used eqs.(13), (14) and $\dot{\rho}_m = -3H\rho_m(1+w_m)$ with $w_m = 0$. The dynamical system of a scalar field with arbitrary potential V was studied in [12], and the critical points with $\dot{x} = 0$ in eq.(19) and constant x are $\dot{\phi} = 0$ and $\ddot{\phi} - xV' = 0$ or equivalently $x = 0, x = q$, respectively. The first case, $\dot{\phi} = 0$, implies $x = 0, w = -1$ and a constant $V(\phi)$ with $\Omega_\phi \rightarrow 1$. At the same time, eq.(20) gives $\dot{y} = -3Hy$ with has a solution $y = y_i(a/a_i)^{-3} \rightarrow 0$ and $\Omega_m \rightarrow 0$. In the second case, $q = \ddot{\phi}/V' = x$, depending on the value of q the quantities $x = q$ and w will take different constant values, and for $w < w_m = 0$ (i.e. $x < 1$) we will have an increasing in $\Omega_\phi \rightarrow 1$ [12]. Setting $q = x$ in eq.(20) we get $\dot{y} = 3Hyw$ with $w = (x-1)/(x+1)$ constant giving a solution

$$y = y_o \left(\frac{a}{a_o}\right)^{3w}. \quad (21)$$

The critical points $\dot{y} = 0$ in eq.(20) are $y = 0$ and $q = 2x - 1$ with y constant. In the first case we have $\Omega_m = 0$ and $\Omega_\phi = 1$ while in the second case eq.(19) becomes $\dot{x} = 3H(1-x)/x$ with a solution

$$e^x(1-x) = e_o^x(1-x_o) \left(\frac{a}{a_o}\right)^{-3}. \quad (22)$$

At large values of a the l.h.s. of eq.(22) vanishes, $x \rightarrow 1, q = 2x - 1 \rightarrow 1, w \rightarrow 0$ and Ω_m, Ω_ϕ are also constant with $\Omega_\phi = 1 - \Omega_m$ (for a generic barotropic fluid the critical point would have been $w \rightarrow w_m$ which in our case is $w_m = 0$). Clearly if $x = q$ at $x = 0$ we are at a critical point $\dot{x} = 0$ in eq.(19) with constant x, w but for $x \neq q, 0$ the system evolves and w is in general not constant.

However, in the present work we do not want to study the critical points but the evolution of x close to present time when the universe is accelerating with x close to zero (w close to -1) but not exactly zero with $\dot{\phi} \neq 0$ and $\ddot{\phi} \neq xV'$. We can assume that $\dot{V} = V'\dot{\phi} < 0$, since we expect ϕ to roll down the minimum of its potential, and the second term in eq.(18) is then negative, while $\ddot{\phi}$ can take either sign. In the region where $\ddot{\phi}/V' < x$ we have $\dot{x} > 0$ while for $\ddot{\phi}/V' > x$ we have $\dot{x} < 0$. If we take what we call a full slow roll defined by $\ddot{\phi} = 0$ and $3H\dot{\phi} = -V'$ then eq.(18) becomes

$$\dot{x} = -(1+z)Hx_z = 6Hx^2 \quad (23)$$

which is positive definite, i.e. $\dot{x} \geq 0, x_z \equiv dx/dz \leq 0$. The solution to eq.(23) is

$$x(a) = \frac{x_o}{1 + 6x_o Ln(a_o/a)} = \frac{x_o}{1 + 6x_o Ln(1+z)}. \quad (24)$$

Therefore if the condition $\ddot{\phi} = 0$ or $|q = \ddot{\phi}/V'| \ll x$ is satisfied eq.(24) gives a decreasing function for x as a function of z and therefore $w(z)$ also decreases. However, we do not expect to be in a full slow roll regime and when

x is small, e.g. $w < -0.9$ one has $x < 0.05$, the slow roll condition $|\ddot{\phi}| < |V'|$ does not imply that $\dot{\phi} \ll xV'$ and the sign of \dot{x} can be positive or negative depending on the sign and size of $q = \ddot{\phi}/V'$ compared to x and x can either grow or decrease. The value of q parameterizes the amount of slow roll of the potential and a full slow roll has $q = 0$ but we expect to be only in an approximate slow roll regime with $|q| \ll 1$ and $A \simeq 1$. We will discuss further the value of q in section IV.

A. Evolution

Taking the differential of eq.(16) we have

$$dx = x_y dy + x_L dL = \frac{-x dy}{1 + 2x + y} + \frac{dL}{6(1 + 2x + y)} \quad (25)$$

where the subscript represents a derivative, e.g. $x_y = dx/dy$, or in terms of derivative w.r.t. the redshift z we have

$$x_z = x_y y_z + x_L L_z \quad (26)$$

with

$$x_y = -\frac{x}{6(1 + 2x + y)} \quad (27)$$

$$x_L = \frac{1}{1 + 2x + y}. \quad (28)$$

Clearly x_L is positive definite while $x_y \leq 0$. The derivative of w is

$$w_z = w_x x_z \quad (29)$$

with

$$w_x = \frac{2x}{(1 + x)^2} \geq 0. \quad (30)$$

In general we can assume that DE redshifts slower than matter, at least for small z , since DE has $w < 0$ and matter $w_m = 0$, so $y = \rho_m/V$ is a growing function of z , i.e. $y_z > 0$. For L constant we see from eq.(26) that $x_z = x_y y_z < 0$ and x will decrease and so will w . On the other hand for y constant we have $x_z = x_y dL$ and an increase on L gives a larger x and w .

We present in the appendix the dynamical equations of L and q and the limits of x from eq.(17) the following: in the limit $L \ll 1$ we have from eq.(16) with

$$x = \frac{L}{6(1 + y)}, \quad w = -1 + \frac{L}{3(1 + y)} \quad (31)$$

and therefore $x \rightarrow 0$, $w \rightarrow -1$ as $L \rightarrow 0$. For $y \gg 1$, with L constant, we have

$$x = \frac{L}{6y}, \quad w = -1 + \frac{L}{3y}, \quad (32)$$

and again we have $x \rightarrow 0$, $w \rightarrow -1$ as $y \rightarrow \infty$. For $y \ll 1$, with L constant, we have

$$x = \frac{L}{6}, \quad w = -1 + \frac{L}{3} \quad (33)$$

giving a constant x and w . For $L \ll 1$, with y constant, we have

$$x = \sqrt{\frac{L}{6}}, \quad w = 1 - 2\sqrt{\frac{6}{L}} \quad (34)$$

giving a constant x and w . Finally, the limit $L/y \rightarrow L_1$ constant with $y \gg 1$ has a constant x and w with

$$x = \frac{L_1}{6}, \quad w = \frac{L_1 - 6}{L_1 + 6} = -1 + \frac{2L_1}{6 + L_1}. \quad (35)$$

As we see from eqs.(32), (33) and (35) all these limits are obtained from eq.(31), i.e. eq.(31) is then valid for the limits $L \ll 1$ or $y \gg 1$ or $y \ll 1$. Clearly depending on the choice of L we can have a decreasing or increasing x, w as a function of redshift.

As we see from eq.(32) for $y \gg 1$ (i.e. $\rho_m \gg V$) valid at larger z we can estimate the evolution of L/y from eq.(20) and (A7) giving

$$\frac{d(L/y)}{dt} = \frac{3HL}{y} \left(\frac{\dot{H}}{3H^2} + \frac{1}{1 + q} \right) \quad (36)$$

and using

$$\frac{\dot{H}}{3H^2} = -\frac{1}{2}(1 + w\Omega_\phi) = -\left(\frac{2x + y}{2(1 + x + y)} \right). \quad (37)$$

we have $d(L/y)/dt > 0$ for $q > (1 + x(1 + 2y + 2x))/(1 + 2y + 3x)$.

1. Late time attractor Solution

The evolution of scalar field has been studied in [12] and the late time attractor for scalar fields leading to an accelerating universe with $\Omega_\phi \rightarrow 1$ requires $w < -1/3$, $L < 2$. In the limit $|\lambda| \gg 1$ one has [12] with $\rho_m \ll \rho_\phi$ and

$$\frac{\dot{\phi}^2}{6H^2} = \frac{L}{6}, \quad \frac{V}{3H^2} = 1 - \frac{L}{6} \quad (38)$$

giving

$$x \equiv \frac{\dot{\phi}^2}{2V} = \frac{L}{6 - L}, \quad w = -1 + \frac{L}{3} \quad (39)$$

If we take the limit $y \ll 1$ and in eq.(31) we recover w in eq.(39). However, for large z we expect y to increase and eq.(39) would not longer be valid. In this region we should use eq.(31) or the full value x from eq.(17) in eq.(8).

III. SCALAR FIELD DE PARAMETRIZATION

In order to have an explicit parametrization of Dark Energy we need to either choose a potential $V(\phi)$ or take a parametrization for L and y . Of course if we want to study a specific potential V we would solve the equation of motion in eq.(5). However, the aim here is to test a wide class of DE models in order to constrain the dynamics of Dark Energy form the observational data. As discussed in sec.IA we will propose an ansatz for L and y that covers the generic behavior of scalar field leading to an accelerating universe.

For the quantity $y = \rho_m/V$ we propose to assume that V redshifts with an EOS $w_o \equiv w(z=0) < 0$, i.e. $V = V_o(a_o/a)^{3(1+w_o)}$, as in eq.(21). This does *not* mean that $\rho_\phi \propto (a_o/a)^{3(1+w_o)}$ since the kinetic energy $\dot{\phi}^2/2$, or equivalently x , may grow faster or slower than V and the EOS for DE $w(z)$ will be in general different than w_o . We have then

$$y = \frac{\rho_m}{V} = y_o \left(\frac{a}{a_o} \right)^{3w_o} = y_o(1+z)^{-3w_o} \quad (40)$$

where $\rho_m = \rho_{mo}(a_o/a)^3$, $y_o = \rho_{mo}/V_o = 2\Omega_{mo}/\Omega_{\phi o}(1-w_o)$ using $V_o = \rho_{\phi o}(1-w_o)/2$. Clearly the function y is an increasing function of z . From eq.(32) we know that as long as L is constant (or growing slower than y) that x and w will decrease as a function of z .

If we want to have a constant EOS for DE at early times $z \gg 1$, as for example matter $w = 0$ or radiation $w = 1/3$, which are reasonable behavior for particles, we should choose L proportional to y for large y or $L/y \rightarrow 0$ if we want $w \rightarrow -1$ at a large redshift. We then propose to take

$$L = L_o + L_1 y^\xi f(z) = L_o + L_1 y_o^\xi \left(\frac{a^{3\xi w_o}}{1 + (a/a_t)^k} \right) \quad (41)$$

where we have chosen

$$f(z) = \frac{1}{1 + (a/a_t)^k} = \frac{1}{1 + [(1+z_t)/(1+z)]^k}, \quad (42)$$

and L_o, L_1, z_t, k are free constant parameters. The function $f(z)$ has as a limit $f(z=0) = [1 + (1+z_t)^k]^{-1}$, $f(z=z_t) = 1/2$ and $f(z \gg z_t) = 1$. The parameters L_o and L_1 give w_o and w at large redshift $z \gg z_t$ while the transition epoch from w_o to $w_1 \equiv w(z \gg z_t)$ is given by z_t and k sets the steepness of the transition. The quantity ξ takes the values $\xi = 1$ and $\xi = 0$ only and we do not consider it as a free parameter but more as two different ansatzes for L .

Using eqs.(26) and (27) we can calculate easily $x_z = x_y y_z x_L L_z$ with

$$L_z = L_y y_z + L_f f_z = L_1(y_z f + y f_z), \quad (43)$$

$L_y = \xi L_1 f y^{\xi-1}$, $L_f = L_1 y^\xi$ and for

$$y_z = -\frac{3\xi w_o y}{1+z}, \quad f_z = \frac{k f^2}{(1+z)} \left(\frac{1+z_t}{1+z} \right)^k \quad (44)$$

Eq.(43) becomes then

$$L_z = \frac{L_1 f y^\xi}{(1+z)} \left(k f \left(\frac{1+z_t}{1+z} \right)^k - 3\xi w_o \right) \quad (45)$$

with $L_1 f y^\xi = L - L_o$, or in terms of the scale factor a we have $L_a = (dz/da)L_z = -L_z(a_o/a)^2$ and

$$L_a = -\frac{L_1 f y^\xi}{a/a_o} \left(k f \left(\frac{a}{a_t} \right)^k - 3\xi w_o \right) \quad (46)$$

For $\xi = 1$ eq.(41) allows a wide class of behaviors for w . If we want w to increase to $w = 0, 1/3$ we would take $L_1 = 6, 12$, respectively, or since in many scalar field models the evolution of w goes from w_o to a region dominated by the kinetic energy density with $w = 1$ and in this case we would should take $L_1 \gg 1$. Of course a $w(z \gg 1) = 1$ would only be valid for a limited period since Ω_ϕ should not dominated the universe at early times. We have included in eq.(41) the case $\xi = 0$ because we want to allow w to increase from w_o at small z and later go to $w \rightarrow -1$ (c.f. pink-dashed line in fig.(3)), since this is the behavior of potentials used as a models of DE as for example $V = V_o \phi^{-n}$, $n = 2/3$ derived from gauge group dynamics [13] where the behavior of $w(z)$ close to present time depends on the initial conditions.

A. Initial Conditions and Free Parameters

Let us summarize the parameters and initial conditions of our parametrization. The EOS w is only a function of x and x is a function of L and y . From eq.(40) we see that y depends on two parameters w_o and $\Omega_{\phi o}$ (or equivalently on y_o, w_o), since we are assuming a flat universe with DE and Matter and $\Omega_{mo} = 1 - \Omega_{\phi o}$. From eqs.(41) and (40) we have the initial conditions at present time as

$$y_o = \frac{\rho_{mo}}{V_o} = \frac{2 \Omega_{mo}}{(1-w_o)\Omega_{\phi o}} \quad (47)$$

$$L_o = \frac{12(1+w_o) + 6y_o(1-w_o^2)}{(1-w_o)^2} - \frac{L_1 y_o^\xi}{1 + (1+z_t)^k}. \quad (48)$$

For $\xi = 1$, from eq.(35), the value of L_1 gives the early time EOS, $w_1 = w(z \gg z_t) = (L_1 - 6)/(L_1 + 6)$ or inverting this expression we have

$$L_1 = \frac{6(w_1 + 1)}{1 - w_1}, \quad (49)$$

giving for example $L_1 = 6$ for $w_1 = 0$, $L_1 = 12$ for $w_1 = 1/3$, $L_1 = 0$ has $w_1 \rightarrow -1$ and $L_1 \gg 1$ has $w_1 \rightarrow 1$. In the case $\xi = 0$ we have the limit $L/y \rightarrow 0$ and $w \rightarrow -1$, independent on the values of L_o, L_1, z_t, k . From eq.(41) we have that L depends on y and L_o, L_1, z_t, k and $w_o, \Omega_{\phi o}$. However, not all parameters are independent,

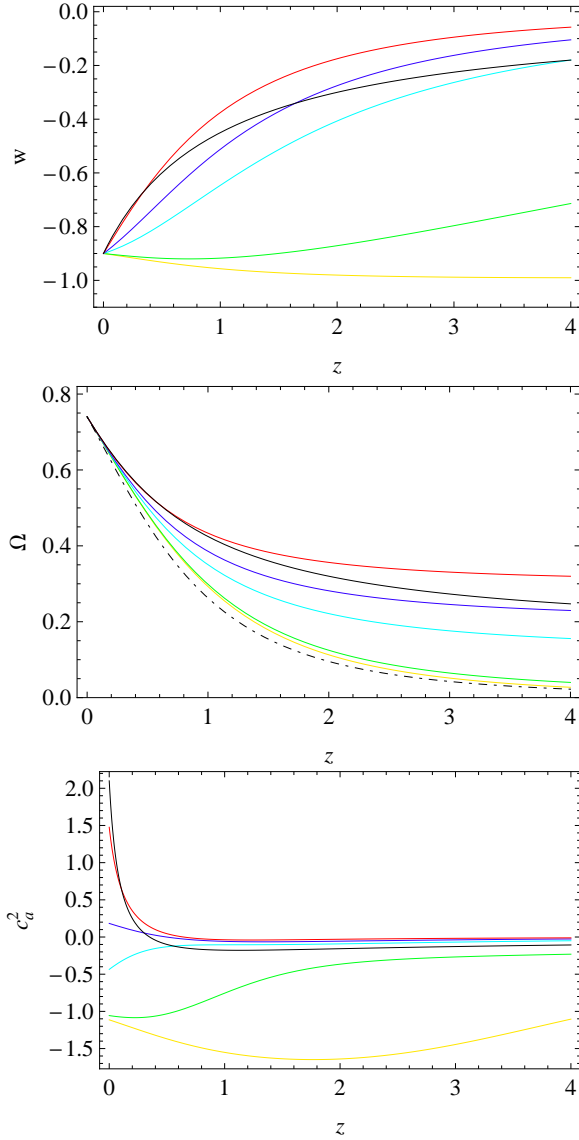


FIG. 1: We show the evolution of Ω_ϕ , w and c_a^2 for different models. We have taken $\xi = 1$, $k = 2$, $L_1 = 6$ with $z_t = 0.1, 1.2, 10, 100$ (red, dark blue, light blue, green and yellow, respectively). In black we have w, Ω_w using w in eq.(50) and Ω_{cc} for a cosmological constant (black dot-dashed). We take in all cases $w_o = -0.9$, $\Omega_{\phi o} = 0.74$ and $L_1 = 6$ giving $w_1 = w(z \gg z_t) = 0$ for large z .

since L_o is a function of $w_o, \Omega_{\phi o}, L_1, z_t, q$ and we are left with $\Omega_{\phi o}$ and four parameters in w . To conclude, the free parameters are $\Omega_{\phi o}$ and for the EOS we can take w_o, w_1 , the transition redshift z_t and the steepness of the transition k .

B. Other Parameterizations

We present here some widely used parameterizations and we compare them with our DE present work. Let us present first a very simple and widely used DE

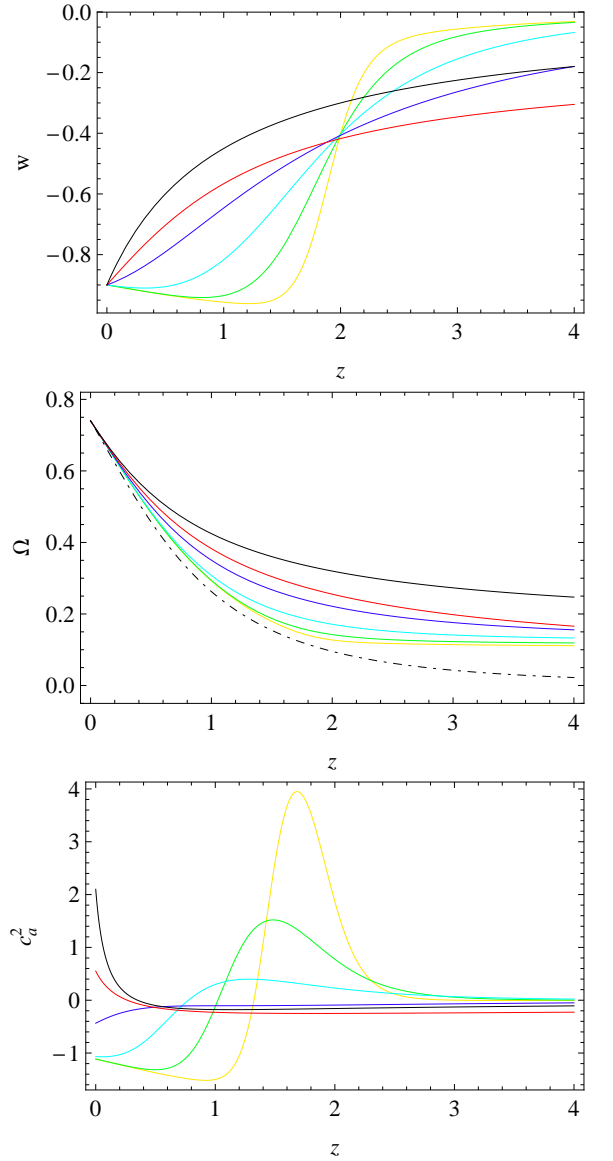


FIG. 2: We show the evolution of Ω_ϕ , w and c_a^2 for different models. We have taken $\xi = 1$, $z_t = 2$, $L_1 = 6$ fixed and $k = 1/2, 2, 5, 10, 20$ (red, dark blue, light blue, green and yellow, respectively). In black we have w, Ω_w using w in eq.(50) and Ω_{cc} for a cosmological constant (black dot-dashed). We take in all cases $w_o = -0.9$, $\Omega_{\phi o} = 0.74$ and $L_1 = 6$ giving $w_1 = w(z \gg z_t) = 0$ for large z .

parametrization [8] given in terms of only two parameters

$$w(a) = w_o + w_a(1 - a) = w_o + w_a \frac{z}{1 + z} \quad (50)$$

with the derivative

$$\frac{dw}{da} = -w_a, \quad \frac{dw}{dz} = \frac{da}{dz} \frac{dw}{da} = (1 + z)^{-2} w_a \quad (51)$$

Clearly w in eq.(50) is convenient since it is a simple EOS and it has only two parameters. However, it may be too restrictive and we do not see a clear connection

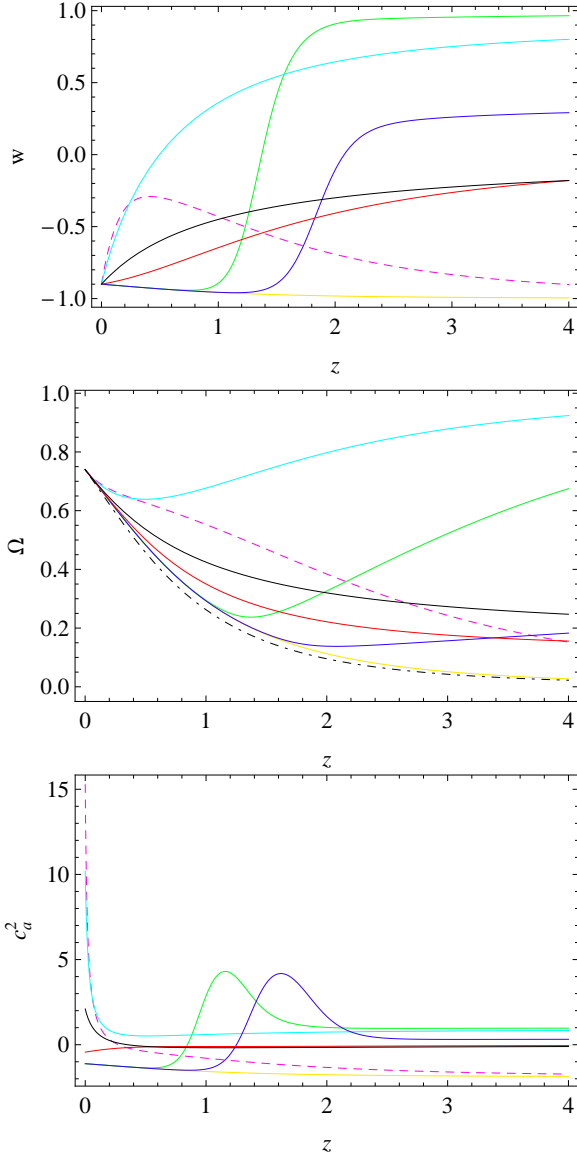


FIG. 3: We show the evolution of Ω_ϕ , w and c_a^2 for different models. We have taken $\xi = 1$, $z_t = 2$ fixed and $(k, L_1) = (2, 6), (20, 12), (2, 100), (20, 1000), (20, 0)$ (red, dark blue, light blue, green and yellow, respectively) and $\xi = 0$ with $z_t = 0.1, k = 10, L_1 = 12$ in pink-dashed line. In black we have w, Ω_w using w in eq.(50) and Ω_{cc} for a cosmological constant (black dot-dashed). We take in all cases $w_o = -0.9$, $\Omega_{\phi o} = 0.74$ and $L_1 = 6$ giving $w_1 = w(z \gg z_t) = 0$ for large z .

between the value of w at small a and its derivative at present time w_a . It has only 3 parameters $\Omega_{\phi o}, w_o, w_a$, two less than our model but our model has a much richer structure.

Another interesting parametrization was presented in [9]. It has 4 free parameters

$$w = w_o + (w_1 - w_o)G, \quad G \equiv \frac{1 + e^{a_d/d}}{1 - e^{1/d}} \frac{1 - e^{(1-a)/d_i}}{1 + e^{(a_d-a)1/d}} \quad (52)$$

where w_o, w_1, a_d, d are constant parameters. The func-

tion G is constraint between $0 \leq \Gamma \leq 1$ with $G = 0$ for $a \gg a_d$ and $G = 1$ for $a \ll a_d$. Therefore a_d is the scale factor where the transition of the EOS w goes from w_o to w_1 . The parameter d gives the width between the transition, for small d the transition from w_o to w_1 is steeper. Even though w in eq.(52) gives a large variety of DE behavior [9], however the sign of the slope is fixed, and therefore our parametrization in eqs.(17) and (41) has a richer structure with the same number of parameters.

In the interesting work of [10] they have followed a similar motivation as in the present work. They have presented a DE parametrization motivated by the dynamics of a scalar field. Their parametrization has either two three parameters in eqs.(25) and (28) in the paper [10], respectively (they do not take a_{eq} as a free parameter but we do think it is an extra parameter). The two parameter involves the quantities $w_i(a \gg a_o)$, which gives the EOS at an early time, and $\lambda(a_{eq}) = V'/V|_{a_{eq}}$ at DM-DE equality (i.e. $\Omega_m = \Omega_{de}$). The second case, the parametrization also involves a term ζ_s (eq.(23) in [10]) which depends on seconde derivative of V and on the value of $\dot{\phi}/H$ at DM-DE equality. Since the functional form of the evolution of the EOS $w(a/a_{eq})$ is fixed in their parametrization the value of w_o at present time is determined if we know the value of a_o/a_{eq} . Therefore, the quantity a_{eq} must also be assumed as a free parameter. As in our present work, they system of equations do not close without the knowledge of the complete V as a function of ϕ . However, since we are both interested in extracting information from the observational data to determine the scalar potential the parametrization given in eqs.(25) and (28) in [10] is a proposal to study a wide range of potentials V . Here we have taken a different parametrization which has a closer connection to the scalar potential $V(\phi)$ given by eqs.(9) and (8).

C. Results

We have plotted w for different sets of the parameters in figs.(1),(2) and (3) to show how w depends on L_o, L_1, z_t and k . We notice that our parametrization in eq.(41) has a very rich structure allowing for w to grow and or decrease at different redshifts. We have also plotted Ω_ϕ and the adiabatic sound speed c_a^2 defined in eq.(61) for each model. We are shoing some extreme cases which do not expect to be observational valid but we want to show the full extend of our parametrization. We have also include a cosmological constant "C.C." (black dot-dashed) in the figures of Ω_ϕ and since $w_{cc} \equiv -1$ and $c_{a,cc}^2 = 0$ we do not include them in the graphs for w, c_a^2 . We also plotted w, Ω_ϕ, c_a^2 (in black) for the parametrization in eq.(50) for comparison. We take in all cases $w_o = -0.9$, $\Omega_{\phi o} = 0.74$.

In fig.(1) we show the evolution of Ω_ϕ, w and c_a^2 for different models. We have taken $\xi = 1, k = 2, L_1 = 6$ with $z_t = 0.1, 1, 2, 10, 100$ (red, dark blue, light blue, green and yellow, respectively). We take in all cases $L_1 =$

6 giving $w_1 = w(z \gg z_t) = 0$. Notice that the yellow line the increase to $w = 0$ does not show in the graph since $z_t = 100$ and we plot w only up to $z = 4$. The slope in w depends on the value of z_t and since $k = 2$ is not large the transition is not too steep. The value of Ω_ϕ decreases slower than a C.C. and for smaller z_t it decreases even more slowly, i.e. when w approaches zero faster.

In fig.(2) we show the evolution of Ω_ϕ , w and c_a^2 for different models. In this case we have taken $\xi = 1, z_t = 2, L_1 = 6$ fixed with $w_1 = 0$ and we vary $k = 1/2, 2, 5, 10, 20$ (red, dark blue, light blue, green and yellow, respectively). We clearly see in the evolution w how the steepness of the transition depends k and that c_a^2 has a bump at z_t and it is more prominent for steeper transition. This is generic behavior and we could expect to see a signature of the transition in large scale structure.

In fig.(3) we show the evolution of Ω_ϕ , w and c_a^2 and we take $\xi = 1, z_t = 2$ fixed and $(k, L_1) = (2, 6), (20, 12), (2, 100), (20, 1000), (20, 0)$ (red, dark blue, light blue, green and yellow, respectively) and $\xi = 0$ with $z_t = 0.1, q = 10, L_1 = 12$ (pink-dashed line). In this case we vary L_1 and we see that for large L_1 the EOS w becomes bigger and it may approach $w \simeq 1$ (e.g. green line). Of course this case is not phenomenologically viable but we plot it to show the distinctive cases of our w parametrization. Once again, a steep transition gives a bump in c_a^2 . The pink-dashed line shows how w can increase at low z and than approach $w = -1$.

We have seen that a our parametrization gives a wide class of w behavior, with increasing and decreasing w . From the observational date we should be able to fix the parameters of L in eq.(41) and we could then have a much better understanding on the underlying potential $V(\phi)$ using $L = (V'/V)^2$.

IV. PERTURBATIONS

Besides the evolution of the homogenous part of Dark Energy $\phi(t)$, its perturbations $\delta\phi(t, x)$ are also an essential ingredient in determining the nature of DE. The formalism we work is the synchronous gauge and the linear perturbations have a line element $ds^2 = a^2(-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j)$, where h is the trace of the metric perturbations [17, 19]. In this sect.(IV) a dot represents derivative with respect to conformal time τ and $H = \dot{a}/a = (da/d\tau)/a$ is the Hubble constant w.r.t. τ , while $H = (da/dt)/a$

A. Scalar Field Perturbations

For a DE given in terms of a scalar field, the evolution requires the knowledge of V and V' while the evolution of $\delta\phi(t, x)$ needs V'' through [17, 19]

$$\delta\ddot{\phi} + 2H\delta\dot{\phi} + [k^2 + a^2V'']\delta\phi = -\frac{1}{2}\dot{h}\dot{\phi}. \quad (53)$$

Eq.(53) can be expressed as a function of a with $\dot{Y} = aHY_a$ for $Y = \delta\phi, \phi, H, h$ and the subscript a means derivative w.r.t. a (i.e. $Y_a \equiv dY/da$), giving

$$\delta\phi_{aa} + \left(\frac{3}{a} + \frac{H_a}{H}\right)\phi_a + \left[\frac{k^2}{a^2H^2} + \frac{V''}{H^2}\right]\delta\phi = -\frac{1}{2}h_a\phi_a. \quad (54)$$

In the slow roll approximation we have

$$\left|\frac{V''}{3H^2}\right| = \Gamma x < 3 \quad (55)$$

where we have used eq.(17) and

$$\Gamma \equiv \frac{V''V}{V'^2}. \quad (56)$$

Eq.(55) implies that an EOS of DE between $-1 \leq w \leq -1/3, 0, 1/3$, with $0 \leq x \leq 1/2, 1, 2$, requires $\Gamma < 3/x = 6, 3, 3/2$, respectively. For a scalar field ϕ to be in the tracking regime one requires Γ to be approximated constant with $\Gamma > 1$ [11]. Therefore the regime $1 < \Gamma < 3/x$ allows a tracking behavior satisfying also the slow roll approximation. Here we are more interested in the late time evolution of DE and the tracking regime is not required and in fact we expect deviations from it. However, if Γ is nearly constant the evolution of the perturbations in (54) are then given only in terms of x and we can use our DE parametrization in eq.(41) and eq.(56) and to calculate them.

We can express the slow roll parameter ϵ, Υ in terms of Γ and L as

$$\epsilon \equiv \frac{1}{2} \left(\frac{V'}{V}\right)^2 = \frac{L}{2}, \quad \Upsilon \equiv \frac{V''}{V} = \Gamma L. \quad (57)$$

We have decided to use L, Υ instead of ϵ, η not to confuse the reader with the inflation parameters and the DE ones.

B. Fluid Perturbations

The evolution of the energy density perturbation $\delta = \delta\rho/\rho$, θ the velocity perturbation [17–20]

$$\begin{aligned} \dot{\delta} = & -(1+w)(k^2 + 9H^2[c_s^2 - c_a^2])\frac{\theta}{k^2} - \frac{\dot{h}}{2} \\ & - 3H(c_s^2 - w)\frac{\delta}{1+w} \end{aligned} \quad (58)$$

$$\dot{\theta} = -H(1 - 3c_s^2)\theta + c_s^2k^2\frac{\delta}{1+w}, \quad (59)$$

and we do not consider an anisotropic stress. The evolution of the perturbations depend on three quantities [19, 20]

$$w = \frac{p}{\rho} \quad (60)$$

$$c_a^2 = \frac{\dot{p}}{\dot{\rho}} = w + \dot{w} \frac{\rho}{\dot{\rho}} \quad (61)$$

$$= w - \frac{\dot{w}}{3H(1+w)} = w + \frac{x_z w_x}{3a(1+w)}$$

$$c_s^2 = \frac{\delta p}{\delta \rho} \quad (62)$$

where w is the EOS, H the Hubble constant in conformal time, c_a^2 is the adiabatic sound speed and c_s^2 is the sound speed in the rest frame of the fluid [18, 19]. For a perfect fluid one has $c_s^2 = c_a^2$ but scalar fields are not perfect fluids. The entropy perturbation G_i for a fluid ρ_i with $\delta_i = \delta \rho_i / \rho_i$ are

$$w_i G_i \equiv (c_{si}^2 - c_{ai}^2) \delta_i = \frac{\dot{p}_i}{\dot{\rho}_i} \left(\frac{\delta p_i}{\dot{p}_i} - \frac{\delta \rho_i}{\dot{\rho}_i} \right) \quad (63)$$

where the quantities G_i and c_{ai}^2 are scale independent and gauge invariant but c_{si}^2 can be neither [17, 20]. In its rest frame a scalar field ϕ with a canonical kinetic term one has $c_s^2 = \delta p / \delta \rho = 1$ [18, 19]. One can relate the rest frame $\hat{\delta}, \hat{\theta}$ to an arbitrary frame δ, θ by [20]

$$\hat{\delta} = \delta + 3H(1+w) \frac{\theta}{k^2} \quad (64)$$

and

$$\delta p = \hat{c}_s^2 \delta \rho + (\hat{c}_s^2 - c_a^2) 3H(1+w) \rho_i \frac{\theta}{k^2} \quad (65)$$

As we see from eqs.(58)-(59) the evolution of δ depends on c_a^2, c_s^2 and w . Using eq.(60) and since w is a function of $x(a)$ we have

$$c_a^2 = w + \frac{x_z w_x}{3a(1+w)} \quad (66)$$

with $w_x/(1+w) = 2/[x(1+x)]$. From eqs.(44) and (45) we can express c_a^2 as a function of the parameters of x .

Finally, we can relate V'' in terms of the adiabatic sound speed c_a^2 in eq.(61) and its time derivative, using $c_a^2 = \dot{p}/\dot{\rho} = 1 + 2V'/3H(d\phi/dt)$, giving

$$\frac{dc_a^2}{dt} = (c_a^2 - 1) \left(\frac{V''}{V'} - \frac{3H}{2} \left(\frac{2d\dot{H}}{3H^2} - (c_a^2 + 1) \right) \right) \quad (67)$$

and for $c_a^2 \neq 1$ we can invert eq.(67) to give

$$\frac{V''}{V'} = \frac{\Gamma V'}{V} = \frac{1}{(c_a^2 - 1)} \frac{dc_a^2}{dt} - \frac{3H}{2} (w_T + c_a^2 + 2) \quad (68)$$

where we have used $\dot{H} = dH/dt = -(\rho_T + p_T)/2 = -3H^2(1+w_T)/2$ with ρ_T, p_T, w_T the total energy density, pressure and EOS, respectively. In our case we have $\rho_T = \rho_m + \rho_\phi$, $p_T = p_m + p_\phi = p_\phi$ and using $\rho_\phi = V(1+x)$ and eqs.(8) and (12) we have

$$w_T \equiv \frac{p_T}{\rho_T} = w\Omega_\phi = \frac{w\rho_\phi}{3H^2} = \frac{w(1+x)}{1+x+y} = \frac{x-1}{1+x+y}. \quad (69)$$

With eq.(69) the l.h.s. of eq.(68) depends then only on y, x and are fully determined by our parametrization. In the full slow roll approximation $\dot{\phi} = 0$ and one has $c_a^2 = -1$.

V. CONCLUSIONS

We have presented a new parametrization of Dark Energy. This parametrization has a rich structure and allows for $w(z)$ to have a wide class of behavior, it may grow and later decrease or other way around. The parametrization of w is given in terms of $x(L, y)$, given in eqs.(27), (41) and (40). The EOS w is constraint between $-1 \leq w \leq 1$ for any value of x , with $0 \leq x$ by definition. The free parameters of w are L_o, L_1, z_t, k , or alternatively w_o and the EOS at an early time $w_1 = w(z \gg z_t)$, given by L_o and L_1 , respectively (c.f. eq.(48), while z_t gives the transition redshift between w_o and w_1 and k sets the steepness of the transition. Besides studying the evolution of Dark Energy we also determined its perturbations from the adiabatic sound speed c_a^2 and c_s^2 given in eqs.(61) and (62), which are functions of x and its derivatives. We have seen that a steep transition has a bump in c_a^2 and this should be detectable in large scale structure.

We can use the parametrization of $x(L, y)$ in eqs.(27), (41) and (40) and c_a^2 and c_s^2 in eqs.(61) and (62) without any reference to the underlying physics, namely the dynamics of the scalar field ϕ , and the parametrization is well defined. However, it is when we interpret $x = \dot{\phi}^2/2V$ and $L = (V'/V)^2 A$ and $y = \rho_m/V$ that we are analyzing the evolution of a scalar field ϕ and we can connect the evolution of w to the potential $V(\phi)$, once the free parameters are phenomenological determined by the cosmological data (only when we take $|q| \gg 1, A \simeq 1$ are we taking in the slow roll approximation).

To conclude, we have proposed a new parametrization of DE which has a rich structure, and the determination of its parameters will help us to understand the nature of Dark Energy.

ACKNOWLEDGMENTS

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Appendix A

The parameter $|q \equiv \ddot{\phi}/V'|$ is clearly smaller than one in the slow roll regime ($\ddot{\phi} < 3H\dot{\phi} \simeq V'$), and let us now determine the dependence of q on the potential $V(\phi)$ and its derivatives. The evolution of q is

$$\dot{q} = \frac{\ddot{\phi}}{V'} - \frac{\ddot{\phi}\dot{\phi}}{V'^2} \quad (A1)$$

$$= 3H \left(-q + (1+q) \frac{\dot{H}}{3H^2} + 2\Gamma x \right) \quad (A2)$$

where we used eq.(14),

$$\frac{\ddot{\phi}}{V'} = -\frac{V''\dot{\phi}}{V'} - \frac{3\dot{H}\dot{\phi}}{V'} - \frac{3H\ddot{\phi}}{V'} \quad (A3)$$

$$= -3Hq + 3H(1+q) \left(\frac{V''}{9H^2} + \frac{\dot{H}}{3H^2} \right), \quad (\text{A4})$$

and

$$\frac{V''}{9H^2} = \frac{\Gamma x}{3}, \quad \Gamma \equiv \frac{V''V}{V'^2}. \quad (\text{A5})$$

We can estimate the value of q if we drop the term proportional to $\ddot{\phi}$ in eq.(A3) and using eq.(37) giving

$$q \simeq \frac{\frac{V''}{9H^2} + \frac{\dot{H}}{3H^2}}{1 - \left(\frac{V''}{9H^2} + \frac{\dot{H}}{3H^2} \right)}. \quad (\text{A6})$$

In a stable evolution of ϕ we have a positive V'' and since \dot{H} is negative both terms have opposite signs, but of course we do not expect a complete cancelation of these terms. However both of them are smaller than one, since $0 \leq -\dot{H}/3H^2 < 1/2$ for $x < 1$ and $|V''/9H^2| = \Gamma x/3 < 1/3$ in the slow roll approximation. A tracker behavior requires $\Gamma > 1$ [11] and $x < 1/\Gamma < 1$. Finally, the evolution of L is given by $\dot{L} = 2\lambda\dot{\lambda}(1+q)^2 + \lambda^2\dot{q}(1+q) =$

$$2L[\dot{\lambda}/\lambda + \dot{q}(1+q)],$$

$$\begin{aligned} \dot{L} &= \frac{12HLx(1-\Gamma)}{(1+q)} + \frac{2L\dot{q}}{(1+q)} = 3HL \left(\frac{2x-q}{1+q} + \frac{\dot{H}}{3H^2} \right) \\ &= 3HL \left(\frac{2(q-x)(1+2x+2y) - y(1+q)}{2(1+q)(1+x+y)} \right). \end{aligned} \quad (\text{A7})$$

We see that at $-1 < q \leq x$ we have $\dot{L} < 0$ giving a decreasing L as a function of time or an increasing L as a function of z . For $(q-x)/(1+q) > y/(1+2x+2y)$ or equivalently for $q > (y+2x(1+2x+2y))/(2+4x+3y)$ we have $\dot{L} > 0$ and a decreasing L as a function of z . The evolution of $d(L/y)/dt$ is given in eq.(36).

Instead of choosing a DE parametrization as in eq.(41) we could solve eqs.(A1) and (A7) for different potentials $V(\phi)$ or by taking different approximated solutions or ansatzes for Γ . However, we choose to parameterize directly L as in eq.(41). Still using $\dot{L} = \dot{a}L_a = aHL_a$ and from eqs.(46), (A7) and $L_1y^\xi f = L - L_o$ we identify

$$(L - L_o) \left(k f \left(\frac{a}{a_t} \right)^k - 3\xi w_o \right) = 3L \left(\frac{2x-q}{1+q} + \frac{\dot{H}}{3H^2} \right) \quad (\text{A8})$$

and the choices of Γ and q would fix the parameters L_o , k and a_t .

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